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FLOW OF A MULTIPHASE MEDIUM OVER A PERMEABLE SURFACE WITH FORMATION OF SEDIMENT

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Flow of a multiphase medium over a permeable surface of arbitrary shape with formation of sediment is considered. A differential equation for determining the sediment-layer thickness has been constructed.

During flow of a multiphase heterogeneous medium over a permeable surface, a continuous phase seeps through a wall, whereas solid particles retained on the flow surface form a sediment layer. Under the conditions of high shear stresses created by the flow of a separable medium and by mass forces, the particles retained on the surface are carried away by the flow. Here the process of filtration occurs without formation of sediment; mathematical simulation of this process was considered in [1, 2]. With the shear stress at the wall being insufficient, filtration occurs with formation of sediment. This process is governed by two phenomena. When the carrying phase filters through a permeable wall, solid particles are retained on the flow surface. The intensity of the increase in the layer thickness is associated only with the rate of filtration; it is independent of the relationship between the densities of the phases, and, therefore, may occur even at $\rho_1 = \rho_i$, $i = 2, \theta$. On the other hand, at considerable relative velocities of the phases, the medium will laminate, which occurs due to the difference between the densities of the phases and does not depend on the presence of the continuous-phase filtration.

The characteristics of individual layers, including the sediment layer, are determined by the physicochemical properties of the constituent phases, characteristic features of the hydrodynamics of the flow, and the intensity of mass forces. The number and composition of disperse phases in individual layers will be different. Therefore, the values of the rheological coefficients and the density of the medium as well as the coefficients of interphase interaction must be determined separately for different layers.

The layer of the deposited mass represents a flux of particles in a state of a sufficiently dense packing capable of creating solid-like structures disintegrating on increase in the intensity of deformation. Even when the material of the grains is undeformable, macrodeformation of the granular skeleton may occur due to the displacement of the grains relative to each other. When considerable shear stresses appear, the sediment formed can flow according to its rheological behavior. Therefore, the mass balance equation for the sediment layer must generally take into account both the scatter of the values for the rate of deposition of individual fractions of a multiphase suspension and spreading of the deposited mass.

To carry out mathematical simulation of the deposited mass, we will consider the latter as a two-phase mixture of a granular solid phase with the liquid that fills the spaces between the grains. In the presence of contacts between dispersed particles, a multiphase mixture is called a contact disperse medium. Momentum transfer in it can occur due to direct interaction between the particles, which is described by the reduced tensor of stresses τ_s. The equations of the momenta of the phases of a contact disperse mixture can be written in the form [3]

$$\rho_{1s} (\mathbf{V}_{1s}\nabla)\, \mathbf{V}_{1s} = -\, \alpha_{1s}\nabla P_s - \mathbf{F}_{12}^{s} + \rho_{1s}\mathbf{F}\,, \tag{1}$$

$$\rho_{2s} (\mathbf{V}_{2s}\nabla)\, \mathbf{V}_{2s} = -\, \alpha_{2s}\nabla P_s + \nabla\tau_2 + \mathbf{F}_{12}^{s} + \rho_{2s}\mathbf{F}\,. \tag{2}$$

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Equation (1) describes the motion of the liquid in the pores of the sediment and Eq. (2) describes macrodeformation of the granular skeleton of the deposited mass. Assuming the smallness of the corresponding rates, the inertia terms can be neglected here.

The force of viscous friction in the contact dispersed phase can be calculated as $\mathbf{F}_{12}^{s} = 4\alpha_{1s}\alpha_{2s}\mu_1 K d^{-2} \times(\mathbf{V}_{1s} - \mathbf{V}_{2s})$, where the coefficient K is determined proceeding from the adopted structure of the medium [3]. The diameter of the particle is unambiguously calculated only for sedimentation of monodispersed particles. Otherwise, a characteristic size of the pores is adopted instead. We note that if the inertia forces and external mass forces are insignificant, Eq. (1) yields the law of filtration in an isotropic medium:

$$V_{1s} - V_{2s} = -\frac{d^2}{4\alpha_{2s}\mu_1 K}\nabla P_s .$$

In order to describe the rheological behavior of a structuring medium, the most universal is the Bulkley–Herschel law:

$$\tau_s = 2\left[\frac{\tau}{\sqrt{2I_2}} + c\left|\sqrt{2I_2}\right|^{s-1}\right]e_{12}\,, \quad \left|\tau_s\right| > \tau\,; \quad e_{12} = 0\,, \quad \left|\tau_s\right| \le \tau\,. \tag{3}$$

Then, after simplifications with account for the slowness of motion, the mass and momentum conservation equations of the contact disperse medium in an orthogonal coordinate system will take the form

$$\frac{\partial\,(H_2H_3\rho_{is}U_{is})}{\partial x_1} + \frac{\partial\,(H_1H_3\rho_{is}V_{is})}{\partial x_2} = 0\,, \quad i = \overline{1, 2}\,; \tag{4}$$

$$-f_{12}^{s}\,(U_{1s} - U_{2s}) + \rho_{1s}F_1 = 0\,; \tag{5}$$

$$-\frac{\alpha_{1s}}{H_2}\frac{\partial P_s}{\partial x_2} - f_{12}^{s}\,(V_{1s} - V_{2s}) + \rho_{1s}F_2 = 0\,; \tag{6}$$

$$\frac{1}{H_1^2H_2H_3}\frac{\partial}{\partial x_2}\left\{H_1^2H_3\left[\tau + c\left(\frac{H_1}{H_2}\frac{\partial\,(U_{2s}/H_1)}{\partial x_2}\right)^s\right]\right\} + f_{12}^{s}\,(U_{1s} - U_{2s}) + \rho_{2s}F_1 = 0\,; \tag{7}$$

$$-\frac{\alpha_{2s}}{H_2}\frac{\partial P_s}{\partial x_2} + f_{12}^{s}\,(V_{1s} - V_{2s}) + \rho_{2s}F_2 = 0\,. \tag{8}$$

These equations are to be solved together with the equations of motion of a multiphase suspension under the following boundary conditions:

$$x_2 = 0: \quad P_s = P_v\,, \quad U_{2s} = 0\,; \tag{9.1}$$

$$x_2 = \delta_s: \quad P = P_s\,, \quad U_i = U_{1s}\delta_1^i + U_{2s}\,(1 - \delta_1^i)\,, \quad \alpha_1V_1 = V_{1s}\,, \quad \tau_1 = \tau\,; \tag{9.2}$$

$$x_2 = h\,(x_1): \quad P = P_a\,, \quad \tau_1 = 0\,. \tag{9.3}$$

The system of equations (4)–(8) on the assumption that $\left|\tau_s\right| > \tau$ is integrated analytically. From Eqs. (5) and (7), subject to the corresponding boundary condition, we find the longitudinal velocities of the motion of the deposited mass and filtrate:

$$U_{2s} = H_1 \int_0^{x_2} \frac{H_2}{H_1} \left[\frac{\tau_1(\delta_s) - \tau}{c} + \frac{1}{cH_1^2 H_3} \int_{x_2}^{\delta_s} \rho_s F_1 H_1^2 H_2 H_3 dx_2 \right]^{1/s} dx_2\,, \quad U_{1s} = U_{2s} + \frac{\rho_{1s} F_1}{f_{12}^{s}}\,.$$

The velocity components of individual phases in the transverse direction can be found from the mass and momentum conservation equations (4), (6), and (8). Subject to the corresponding boundary conditions, we obtain

$$V_{2s} = -\frac{1}{H_1 H_3} \int_0^{x_2} \frac{\partial}{\partial x_1} (H_2 H_3 U_{2s})\, dx_2\,, \quad V_{1s} = V_{2s} - \frac{\alpha_{1s}}{f_{12}^{s}} \left(\frac{1}{H_2} \frac{\partial P_s}{\partial x_2} - \rho_1^0 F_2 \right).$$

It is advisable to select the technique of determining the value of $\partial P_s / \partial x_2$ in the latter equation after the motive forces of the filtration process have been analyzed. When filtration occurs due to the mass force, the pressure gradient can be found from (6) and (8) in the form $\partial P_s / \partial x_2 = H_2 \rho_s F_2$. If the motive force of the filtration process is the difference in the pressures between the regions located on different sides of the permeable barrier, the expressions for the velocities U_{1s} and V_{1s} are substituted into Eq. (4), the integration of which yields the value of $\partial P_s / \partial x_2$.

Solutions of the equations of motion of a suspension can be obtained by the method of the surfaces of equal flow rates [4]. The transformed equations of motion over the radial coordinate that are written on the stream lines for the case of a multiphase medium will take the form [1, 2]

$$\frac{\rho_1 U_1^k}{H_1} \frac{dU_1^k}{dx_1} = -\frac{\alpha_1}{H_1} \frac{dP^k}{dx_1} + \alpha_1 J^k (x_1, x_2) \frac{dy_1^k}{H_1 dx_1} - \frac{\rho_1 U_1^k V_1^k}{H_1 H_2} \frac{\partial H_1}{\partial x_2} +$$

$$+ \frac{1}{H_1^2 H_2 H_3} \frac{\partial}{\partial x_2} \left[H_1^2 H_3 m \left| \frac{H_1}{H_2} \frac{\partial}{\partial x_2} \left(\frac{U_1^k}{H_1} \right) \right|^{n-1} \frac{H_1}{H_2} \frac{\partial}{\partial x_2} \left(\frac{U_1^k}{H_1} \right) \right] - \sum_{j=2}^{\theta} F_{1j1} + \rho_1 F_1\,, \quad k = \overline{2, N_1}\,; \tag{10}$$

$$\frac{\rho_i U_i^l}{H_1} \frac{dU_i^l}{dx_1} = -\frac{\alpha_i}{H_1} \frac{dP^l}{dx_1} + \alpha_i J^l (x_1, x_2) \frac{dy_i^l}{H_1 dx_1} - \frac{\rho_i U_i^l V_i^l}{H_1 H_2} \frac{\partial H_1}{\partial x_2} + \sum_{\substack{j=1 \\ i \neq j}}^{\theta} F_{ij1} + \rho_i F_1\,, \quad i = \overline{2, \theta}\,, \quad l = \overline{2, N_i}\,. \tag{11}$$

The positions of the surfaces of equal flow rates for individual phases can be found from the system of differential equations

$$\frac{dy_1^k}{dx_1} = \frac{dy_1^{k-1}}{dx_1} + \frac{2H_1 Z \alpha_1 V_1 (\delta_s)}{\Delta_1^k} \delta_2^k - \frac{y_1^k - y_1^{k-1}}{\Delta_1^k} \frac{d\Delta_1^k}{dx_1}\,, \quad k = \overline{2, N_1}\,;$$

$$\frac{dy_i^l}{dx_1} = \frac{dy_i^{l-1}}{dx_1} + \frac{2H_1 Z \alpha_i V_i (\delta_s)}{\Delta_i^l} \delta_2^l - \frac{y_i^l - y_i^{l-1}}{\Delta_i^l} \frac{d\Delta_i^l}{dx_1}\,, \quad i = \overline{2, \theta}\,, \quad l = \overline{2, N_i}\,, \tag{12}$$

where $\Delta_i^j = (H_2 Z \alpha_i U_i)^{j-1} + (H_2 Z \alpha_i U_i)^j$; $Z = H_3 (x_{3f} - x_{3in})$. Since the surfaces of equal flow rates and their numbers for each component of the multiphase medium are introduced individually, in system (12) the equations for the stream lines of the dispersion phase and disperse inclusions are written down separately for clarity. The characteristic features of integration of the system of recurrent equations (10)–(12) and the technique of calculation of the pressure gradient dP^k / dx_1 are given in [1, 2].

The rate of sedimentation of the fractions must be determined from the projection of the equations of motion of the separated multiphase medium layer to the x_2 axis [1, 2]:

$$-\rho_i \frac{U_i^2}{H_1 H_2}\frac{\partial H_1}{\partial x_2} = -\frac{\alpha_i}{H_2}\frac{\partial P}{\partial x_2} + (1 - 2\delta_1^i)\sum_{\substack{j=1\\ i\neq j}}^{\theta} F_{ij2} + \rho_i F_2\,, \quad i = \overline{1, \theta}\,.$$

The latter system of equations is easily reduced to the form

$$\sum_{\substack{j=1\\ i\neq j}}^{\theta} f_{ij}\,(V_i - V_j)^2 = -\frac{\rho_i U_i^2}{H_1 H_2}\frac{\partial H_1}{\partial x_2} + \frac{\alpha_i}{H_2} J\,(x_1, x_2) - \rho_i F_2\,, \quad i = \overline{2, \theta}\,, \tag{13}$$

where $J(x_1,\ x_2)\ =\ \rho F_2 H_2 + \sum_{i=1}^{\theta} \rho_i U_i^2 \frac{\partial H_1}{H_1 \partial x_2}$. When $\mathrm{Re}_{12} << 1$, the interphase interaction force has a linear structure [3]; therefore system (13) can be linearized having replaced the quantity $f_{ij}(V_i - V_j)^2$ on the left-hand side of the equations by $f_{ij}(V_i - V_j)$. The velocity V_1 is determined separately with account for the rate of filtration; therefore in solving the system of algebraic equations (13) it is considered known.

It should be noted that rheological law (3) allows one to consider all the possible variants of the behavior of the deposited mass. To consider them, we will integrate the sum of equations (5) and (7) subject to boundary condition (9.2):

$$c\left(\frac{H_1}{H_2}\frac{\partial\,(U_{2s}/H_1)}{\partial x_2}\right)^s = \frac{1}{H_1^2 H_3}\int_{x_2}^{\delta_s} H_1^2 H_2 H_3 \rho_s F_1 dx_2 + \tau_1\,(\delta_s) - \tau\,. \tag{14}$$

When for any $x_2 \le \delta_s$ the inequality below is satisfied

$$\frac{1}{H_1^2 H_3}\int_{x_2}^{\delta_s} H_1^2 H_2 H_3 \rho_s F_1 dx_2 + \tau_1\,(\delta_s) > \tau\,, \tag{15}$$

the sediment will flow as a whole, i.e., in the same way as the power-law fluid with the rheological constants c and s under the action of a shear stress equal to $\tau_s - \tau$. When inequality (15) is satisfied only in the definite interval $0 < x_2 \le x_2^*$, the so-called flow core appears in the layer. The part of the sediment layer of thickness $x_2^* = \delta_s - \delta_*$ adjoining the wall will flow with the velocity changing over the cross section, whereas the outer part of the layer of thickness δ_* will move as a solid body with the velocity equal to the velocity at the boundary between the layers. If the sum of the first two terms on the right-hand side of (14) is smaller than τ, the motion of the sediment does not occur.

We will construct the mass balance equation for the sediment layer; it takes into account the difference between the rates of deposition of the multiphase suspension fractions. We will consider an integral relation of conservation of the amount of the solid phase in an elementary volume of the sediment for the time $\Delta t = t - t_{\mathrm{in}}$ (Fig. 1). At any time t this elementary volume is calculated from the equation

$$\Omega\,(t) = H_3 \Delta x_3 \int_{x_1}^{x_1+\Delta x_1} \delta_s\,(x_1, t)\,H_1 dx_1\,. \tag{16}$$

The change in the amount of solid particles in the considered volume is determined by two factors: the flux of depositing particles Ω_{flux} and the flow of the deposited mass Ω_{flow} in the direction of the x_1 axis:

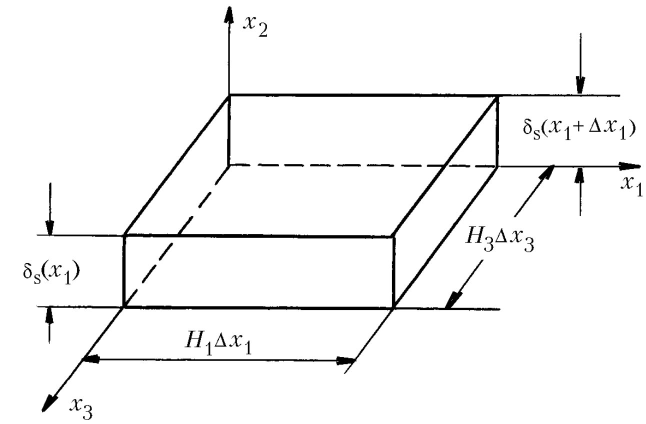

Fig. 1. Elementary volume of the sediment.

$$\alpha_{2s}\,(\Omega\,(t) - \Omega\,(t_{in})) = \Omega_{flux} + \Omega_{flow}\,. \tag{17}$$

The area of the upper face of the elementary volume is equal to $H_3\Delta x_3 L$; consequently, the number of particles deposited for the time Δt can be calculated in terms of the integral

$$\Omega_{flux} = H_3\Delta x_3 \int_{t_{in}}^{t} L\Omega_V\,(x_1,\,\delta_s)\,dt\,, \tag{18}$$

where $\Omega_V(x_1,\ \delta_s) = \sum_{i=2}^{\theta} \alpha_i V_i(x_1,\ \delta_s)$ is the flux of solid particles to the wall caused, in the first place, by the filtering motion of the liquid through the sediment layer and, second, by the difference between the velocities of different phases in the suspension layer. The length of the arc L is calculated from the equation

$$L = \int_{x_1}^{x_1+\Delta x_1} \sqrt{1 + \left(\frac{H_2 d\delta_s}{H_1 dx_1}\right)^2}\; H_1 dx_1\,. \tag{19}$$

The change in the number of particles in an elementary volume because of the motion of the sediment is determined by the difference between the inflowing and flowing-out mass for the time interval considered:

$$\Omega_{flow} = H_3\Delta x_3 \int_{t_{in}}^{t} \int_{0}^{\delta_s(x_1,t)} \alpha_{2s} U_{2s}\,(x_1,\,t)\,H_2 dx_2 dt - H_3\Delta x_3 \int_{t_{in}}^{t} \int_{0}^{\delta_s(x_1+\Delta x_1,t)} \alpha_{2s} U_{2s}\,(x_1 + \Delta x_1,\,t)\,H_2 dx_2 dt\,.$$

We introduce the notation

$$Q_s = \int_{0}^{\delta_s(x_1,t)} U_{2s}\,(x_1,\,t)\,H_2 dx_2$$

and simplify the latter relation:

$$\Omega_{flow} = \alpha_{2s} H_3\Delta x_3 \int_{t_{in}}^{t} (Q_s\,(x_1,\,t) - Q_s\,(x_1 + \Delta x_1,\,t))\,dt\,. \tag{20}$$

For the small interval Δx_1, integrals (16) and (19) can be approximated by the trapezium equation. Further, after substitution of Eqs. (16), (18), and (20) into Eq. (17), we will divide it by $\alpha_{2s} H_1 \Delta x_1 H_3 \Delta x_3$ and perform the limiting transition $\Delta x_1 \to 0$:

$$\delta_s (x_1, t) - \delta_s (x_1, t_{in}) = \int\limits_{t_{in}}^{t} \sqrt{1 + \left(\frac{H_2 d\delta_s}{H_1 dx_1}\right)^2} \frac{\Omega_V (x_1, \delta_s)}{\alpha_{2s}} dt - \int\limits_{t_{in}}^{t} \frac{1}{H_1} \frac{\partial Q_s}{\partial x_1} dt \, .$$

Differentiating the latter relation with respect to t, we obtain the mass balance equation for the sediment layer:

$$\frac{\partial \delta_s}{\partial t} + \frac{\partial Q_s}{H_1 \partial x_1} = \sqrt{1 + \left(\frac{H_2 d\delta_s}{H_1 dx_1}\right)^2} \frac{1}{\alpha_{2s}} \sum_{i=2}^{\theta} \alpha_i V_i (x_1, \delta_s) \, , \tag{21}$$

which makes it possible to take into account the difference between the rates of deposition of the fractions in a multiphase separable medium.

When the rate-induced nonequilibrium state of the phases is insignificant and the development of the sediment is determined only by the filtration motion of the liquid through the sediment, we may consider that $V_i \approx V_1 = V_{1s}(x_1, \delta_s)/\alpha_1$. Then, provided the condition $(H_2/H_1)^2 (d\delta_s/dx_1)^2 << 1$ is satisfied, dependence (21) yields the generally adopted mass balance equation of the sediment for a monodisperse suspension [5]:

$$\frac{d\delta_s}{dt} + \frac{\partial Q_s}{H_1 \partial x_1} = \frac{\alpha}{\alpha_1 \alpha_{2s}} V_{1s} (x_1, \delta_s) \, . \tag{22}$$

In practice, the regime of filtration is most often implemented with formation of the sediment that is motionless relative to the wall. Here, two cases are possible. In some apparatus, for example, in band-type filters or in drum vacuum filters with the outer working surface, the permeable wall and, consequently, the sediment layer are continuously carried away from the zone of filtration. In this case, the thickness of the sediment is independent of time. When $U_{2s} = W = \text{const}$, Eqs. (21) and (22) yield respectively the differential equations

$$\frac{d\delta_s}{dx_1} = \frac{H_1 \Omega_V}{H_2 \sqrt{(\alpha_{2s} W)^2 - \Omega_V^2}} \, , \tag{23}$$

$$\frac{d\delta_s}{dx_1} = \frac{H_1}{H_2} \frac{\alpha}{\alpha_1 \alpha_{2s}} \frac{V_{1s} (x_1, \delta_s)}{W} \, . \tag{24}$$

When the particles are accumulated on an immobile wall, the velocity U_{2s} and flow rate Q_s are equal to zero. The thickness of the sediment will change, however, both in time and over the longitudinal coordinate. The dependence of the sediment thickness on the coordinate x_1 will manifest itself in the presence of lamination of phases when some of the fractions settle out before others. Then the mass balance equations of the sediment (21) and (22) will take on the form

$$\frac{d\delta_s}{\partial t} = \frac{\Omega_V}{\alpha_{2s}} \sqrt{1 + \left(\frac{H_2 d\delta_s}{H_1 dx_1}\right)^2} \, , \tag{25}$$

$$\frac{d\delta_s}{dt} = \frac{\alpha}{\alpha_1 \alpha_{2s}} V_{1s} (x_1, \delta_s) \, . \tag{26}$$

Such a change in the thickness in time will proceed until it attains the level δ_* which is the maximum thickness of the sediment layer at which there is no flow. When $\delta_s > \delta_*$, the layer begins to fall down the wall.

In the case where the layer develops according to one of the equations (23)–(26), the conservation equations (4)–(6) are transformed into ordinary filtration equations, the solution of which presents no difficulties. Thus, calculation of the process of filtration with formation of a sediment can be brought to numerical integration of a system of ordinary differential equations (10)–(12) together with one of the equations (21)–(26). In the case where the sediment layer develops according to one of the equations (25) and (26), numerical calculations become more involved because of the time derivative. Then, the following cyclic algorithm can be used:

1) the distribution of the positions of the surfaces of equal flow rates y_i^k and the velocity profile U_i^k at the inlet section are selected;

2) at the initial time instant $t = t_{in}$ there is no sediment, $\delta_s = 0$;

3) the system of equations (10)–(12) is solved numerically for $y_i^k(x_1)$ and $U_i^k(x_1)$;

4) the filtration rate $V_{1s}(x_1)$ is calculated from the determined distribution of the characteristics of the suspension layer;

5) Eq. (25) or (26) yields the change in the sediment thickness over the surface at the time instant $t + \Delta t$;

6) the values of the sediment layer thickness are smoothed, for example, by the equation $\bar{\delta}_s(x_1) = [\xi\delta_s(x_1 - \Delta x_1) + (1 - \xi)\delta_s(x_1) + \xi\delta_s(x_1 + \Delta x_1)]/(1 + \xi)$ and the derivatives $d\bar{\delta}_s/dx_1$ are calculated;

7) stages 3–6 are repeated.

In the case of a multiphase or polydisperse medium the number of equations in system (10)–(12) is increased, which complicates the numerical calculations. Moreover, equations for calculating the effective viscosity and force of interphase interaction are lacking for such media. Therefore, in order to simplify the problem, average characteristics of particles are often used, thus replacing the polydisperse composition of the disperse phase by a monodisperse one. Depending on the importance of the characteristics of particles for a specific process, a mean diameter can be introduced, which is based on different attributes. To calculate the average size, the following equation can be used:

$$d = \left(\sum_i \sigma_i d_i^a \Big/ \sum_i \sigma_i d_i^b \right)^{1/(a-b)} . \qquad (27)$$

Then, for example, imparting the values 1 and 0, 2 and 0, 3 and 0, and 3 and 2 to the exponents a and b, we obtain the mean diameter based on the size, surface, mass, and volume-surface attribute, respectively.

To increase the accuracy of approximation (27), it is sometimes advisable to average the characteristics of the disperse phase not over the entire spectrum of particles, but rather over a certain number of its fractions, each of which is characterized by its own equivalent size and by other macrocharacteristics. The use of this way of simplification makes it possible to consider heterogeneous media as two-phase or three-phase ones. For example, application of filtering auxiliary substances in separation of a finely dispersed suspension is well known. For the particles of the auxiliary substance and dispersed phase of the suspension one may introduce individual averaged diameters of the particles, using for the purpose Eq. (27). The process of separation is calculated using as a basis the knowledge of the hydrodynamics of a three-phase system with particles of two species.

As another example, we will consider the case of a suspension containing coarsely dispersed particles in a suspension of particles of colloidal dispersivity. A mixture of a colloidal fraction and a continuous medium can be considered as a certain homogeneous medium into which the particles of the coarse fractions were placed. Since a suspension of highly dispersed particles is structurized and, therefore is a Newtonian one, we may conclude that in this case the model of a suspension of particles in a non-Newtonian medium is applicable [6]. If the mean diameter of coarsely dispersed particles is determined according to (27), we arrive at the case of a two-phase medium with a non-Newtonian dispersion phase.

The rheology and force of interaction between the phases for two-phase and three-phase (bidisperse) media have been studied rather adequately. Some of the results and citings of other publications can be found, for example, in [6–10].

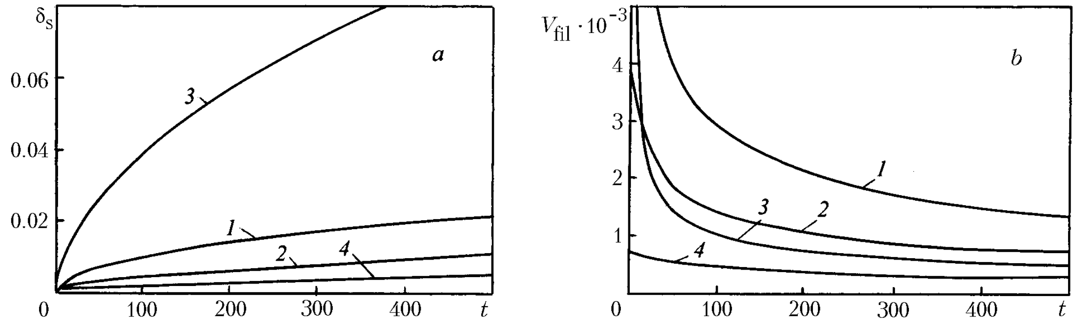

Fig. 2. Changes in the thickness of the sediment layer (a) and rates of filtration in time (b): 1) ΔP = 30,000 N/m^2, μ_1 = 0.01 Pa·sec, and α_2 = 0.01; 2) 7500, 0.01, and 0.01; 3) 30,000, 0.01, and 0.11; 4) 30,000, 0.05, and 0.01.

Numerical calculations are performed after the flow region has been selected and the Lamé coefficients determined. Specific definition of the resulting equations and computational relations does not present difficulties once the type of filtering apparatus has been selected. Figure 2 presents some numerical calculations of the process of filtration of a suspension during its flow over an inclined permeable plane. The angle of inclination to the horizontal line is 60°. The thickness of the sediment was calculated from Eq. (26). It was assumed that the influence of the excess pressure $\Delta P = P - P_v$ on the rate of filtration dominates over the influence of the mass force. As is seen from Fig. 2a, the thickness of the sediment increases monotonically with time. The intensity of this increase is directly proportional to the pressure difference (curves 1 and 2), and the concentration of the suspension (curves 1 and 3) is inversely proportional to the carrying-phase viscosity (curves 1 and 4). In all cases, the intensity of the increase decreases as a result of the decrease in the rate of filtration. Figure 2b shows the effect of the same parameters on a value of the filtration rate. The rate of filtration is decreased with increase in the medium viscosity (curves 1 and 4) and concentration of particles (curves 1 and 3), whereas it increases with the pressure difference (curves 1 and 2).

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NOTATION

c, coefficient of the consistency of a deposited mass, kg·sec^{s-2}/m; d, mean diameter of a particle, m; d_i, diameter of a particle of the ith fraction, m; e_{12}, tensor of deformation rates; f_{12} and f_{12}^{s}, coefficients of the force of interphase interaction in a suspension and a contact disperse medium; $\mathbf{F}(F_1, F_2)$, vector of the acceleration of mass forces (and its components in the direction of the x_1 and x_2 axes), m/sec^2; $\mathbf{F}_{ij}$, F_{ijk} (k = 1, 2, 3), vector of the force of interphase interaction and its projections onto the corresponding x_k axes, kg/(m^2·sec^2); $\mathbf{F}_{12}^{s}$, vector of the force of interphase interaction in a contact disperse medium, kg/(m^2·sec^2); h, thickness of the mixture film, m; H_i, Lamé coefficients; I_2, quadratic invariant of the tensor of deformation rates; K, coefficient of the force of viscous friction; L, length of the arc, m; m, coefficient of the consistency of a heterogeneous medium, kg·sec^{n-2}/m; n, s, coefficients of nonlinearity; N_i, number of the surfaces of equal flow rates for the ith phase; P_s, P_a, P_v, and P, pressure in the pores of the sediment, atmospheric pressure, pressure behind a porous wall, and pressure in the liquid film, respectively, N/m^2; Q_s, flux of the sediment, m^2/sec; t, time, sec; U_i, V_i, velocity components of the ith phase of a suspension in the direction of the x_1 and x_2 axes, m/sec; $\mathbf{V}_{is}(U_{is}, V_{is})$, vector of the velocity of a flowing sediment (and its components in the direction of the x_1 and x_2 axes), m/sec; W, velocity of the motion of a surface, m/sec; V_{fil}, rate of filtration, m/sec; x_i, orthogonal coordinates; α_i, volumetric concentration of the ith phase; $\alpha = \sum_{i=2}^{\theta} \alpha_i$; α_{1s}, porosity of the sediment ($\alpha_{1s} = 1 - \alpha_{2s}$); δ_s, sediment thickness, m; δ_1^k, delta-function; μ_1, velocity of the carrying phase of the suspension, Pa·sec; θ, number of phases; ρ_i^0, density of the ith phase ($\rho_i = \alpha_i\rho_i^0$, $\rho_{is} = \alpha_{is}\rho_i^0$, $\rho_s = \rho_{1s} + \rho_{2s}$, $\rho = \sum_{i=1}^{\theta} \rho_i$, kg/m^3; σ_i, number of particles of the ith fraction per unit volume; τ, τ_1, and τ_3, limiting shear stress and shear tensor for suspension and sediment layers, N/m^2; ω_{1i}, relative velocity of the motion of phases, m/sec; ξ, smoothing coefficient; Ω, Ω_{flux}, and Ω_{flow}, elementary volume, its changes because of the flux of depositing particles, and flow of the sediment, m^3; Ω_V, velocity of a flux of particles, m/sec; $\text{Re}_{12} = \rho_1^0 d\omega_{1i}/\mu_1$, Reynolds numbers. Subscripts: a, atmospheric; v, behind a permeable wall (vacuum); in, initial value; f, final value; fil, filtration (at the sediment–medium boundary); p, sediment; V, flow velocity; i, number of a phase or fraction; k and l, numbers of the surfaces of equal flow rates for a continuous and a dispersed phase; s, sediment. Superscripts: *, characteristic value; s, sediment.

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